MBF-003-001205 Seat No. ________

B. Sc. (Sem. II) (CBCS) Examination

March / April - 2018

Mathematics : BSMT-201(A)

(Geometry, Trigonometry & Matrix Algebra)

[Old Course]

Faculty Code : 003

Subject Code : 001205

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instruction : (i) All questions are compulsory.

(ii) Question 1 contains 20 short questions of one mark each.

(iii) Question 2 and 3 carry 25 marks each with internal choices.

1 Answer the following questions : **20**

(1) Write equation of right circular cylinder whose axis is parallel to Y-axis and radius is r.

(2) Write definition of right circular cylinder.

(3) Define singular matrix.

(4) If $A = \begin{bmatrix} 2 & -1 \\ -2 & 4 \end{bmatrix}$, then find A^{-1}.

(5) Define unitary matrix.

(6) Define Cauchy sequence.

(7) Define Rank of a matrix.

(8) Define Oscillatory sequence.

(9) Define Hermitian Matrix.

(10) Define Orthogonal Matrix.

(11) Prove that $\sin 2x = 2 \sin x \cos x$.

(12) $\cosh^2 x - \sinh^2 x =$_______.

(13) Find read and imaginary part of e^{5+4i}.

(14) State D'moirve's theorem.

(15) Write the formula of $\sin x$ in power of x.

(16) Write the expansion of $\cos^n \theta$ in cosine function.

(17) Prove that $\cos 3\theta = 4 \cos^3 \theta - 3 \cos \theta$.

(18) If $z = \cos \theta + i \sin \theta$ then prove that

$$z^2 + \frac{1}{z^2} = 2 \cos 2\theta.$$

(19) Find real and imaginary part of e^{z^2}.

(20) Simplify $\dfrac{(\cos \theta + i \sin \theta)^2}{(\cos \theta - i \sin \theta)^4}$.

2 (a) Answer any three out of six : **6**

(1) Prove that $A = \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}$ is a nilpotent matrix of index 2.

(2) If A and B are Hermitian matrices then prove that $A + B$ is also Hermitian.

(3) Prove that $A = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & 3 & -4 \end{bmatrix}$ is an idempotent matrix.

(4) Find rank of a matrix $A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}$.

(5) Find eigen value of matrix $A = \begin{bmatrix} 5 & 3 \\ -1 & 1 \end{bmatrix}$.

(6) Discuss the convergence of a sequence $\left\{\sqrt{n+1} - \sqrt{n}\right\}$.

(b) Answer any three out of six : **9**

(1) Find the equation of cylinder whose generator is parallel to $\frac{x}{2} = \frac{y}{3} = \frac{z}{4}$ and passing though $x^2 + xy + y^2 = 1,\ z = 0$.

(2) If $A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ find adj(A).

(3) Find inverse of $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ using preliminary row operation.

(4) Prove that Eigen values of Hermitian Matrix are real numbers.

(5) Verify Cayley Hamilton Theorem for matrix

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}.$$

(6) Prove that the sequence $\{S_n\}$ is convergent, where

$$S_n = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \ldots.. + \frac{1}{n!}$$

(c) Answer any two out of five : **10**

(1) Find equation of cylinder whose generator is parallel to $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$ and enveloping curve is $x^2 + y^2 + z^2 = a^2$.

(2) State and prove Caylay-Hamilton Theorem.

(3) Solve the system of linear equation :

$$x + y + z = 9,$$
$$2x + 5y + 7z = 52,$$
$$2x + y - z = 0.$$

(4) If $\lim_{n \to \infty} a_n = l$ then prove that

$$\lim_{n \to \infty} \left(\frac{a_1 + a_2 + \ldots\ldots\ldots\ldots + a_n}{n} \right) = l.$$

(5) Prove that every square matrix can be uniquely express as sum of a symmetric and skew symmetric matrices.

3 (a) Answer any three out of six : **6**

(1) Prove that $\cos 7\theta = \cos^7 \theta - 21 \cos^5 \theta \sin^2 \theta + 35 \cos^3 \theta \sin^4 \theta - 7 \cos \theta \sin^4 \theta$.

(2) Express $\tan 7\theta$ in term of $\tan \theta$.

(3) Express $\sin^3\theta$ in sine function.

(4) Find the value of $\log(1-i)$.

(5) Solve $7\sinh x + 20\cosh x = 24$.

(6) Prove that $\dfrac{1+\tanh\theta}{1-\tanh\theta} = \cosh 2\theta + \sinh 2\theta$.

(b) Answer any three out of six : **9**

(1) If $\tan\theta = \dfrac{1}{2}$, then find the value of $\tan 6\theta$.

(2) Show that

$$\cos^3 x = \frac{1}{4}\left[4 - \frac{x^2}{2!}\left(3^2+3\right) + \frac{x^4}{4!}\left(3^4+3\right) - .....\right]$$

(3) Solve $x^3 + 1 = 0$.

(4) Solve $\cosh z = \dfrac{1}{2}$.

(5) Prove that $\cosh^{-1} x = \log\left(x + \sqrt{x^2-1}\right)$.

(6) Find $\log\left(-\dfrac{1}{2} - i\dfrac{\sqrt{3}}{2}\right)$.

(c) Answer any two out of five : **10**

(1) Prove that

$$2048\ \sin^7\theta\cos^5\theta = 20\sin 2\theta + 5\sin 4\theta$$
$$-10\sin 6\theta + 4\sin 8\theta + 2\sin 10\theta - \sin 12\theta.$$

(2) If α and β are roots of equation

$x^2 - 2x + 4 = 0$ then prove that

$$\alpha^n + \beta^n = 2^{n+1}\cos\left(\frac{n\pi}{3}\right).$$

(3) Prove that

$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$, where n is an integer.

(4) Prove that $\sinh^{-1}(\tan\theta) = \log\tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)$.

(5) Find real and imaginary part of $\tan^{-1}(x + iy)$.